Section 2: Equations and Inequalities

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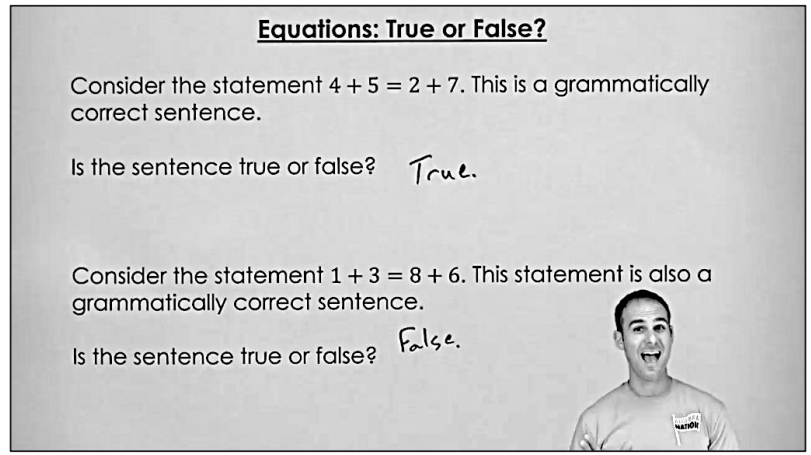

The following Mathematics Florida Standards will be covered in this section:
A-APR.2.3 - Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.
A-CED.1.1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
A-CED.1.2 - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
A-CED.1.4 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.
A-REI.1.1 - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-REI.2.3 - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
A-REI.4.10 - Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.1.2 (honors standard) - Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Section 2: Equations and Inequalities

Section 2 – Topic 1
Equations: True or False?

Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence.

Is the sentence true or false?

Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence.

Is the sentence true or false?

The previous statements are examples of ***number sentences***.

- A number sentence is a statement of equality between two _____ expressions.
- A number sentence is said to be true if both numerical expressions are _____.
- If both numerical expressions don't equal the same number, we say the number sentence is _____.
- True and false statements are called ***truth values***.

Let's Practice!

1. Determine whether the following number sentences are true or false. Justify your answer.

   a. $13 + 4 = 7 + 11$

   b. $\frac{1}{2} + \frac{5}{8} = 1.4 - 0.275$

Try It!

2. Determine whether the following number sentences are true or false. Justify your answer.

   a. $(83 \cdot 401) \cdot 638 = 401 \cdot (638 \cdot 83)$

   b. $(6 + 4)^2 = 6^2 + 4^2$

A number sentence is an example of an ***algebraic equation***.

- An algebraic equation is a statement of equality between two _____.
- Algebraic equations can be number sentences (when both expressions contain only numbers), but often they contain _____ whose values have not been determined.

Consider the algebraic equation $4(x + 2) = 4x + 8$.

Are the expressions on each side of the equal sign equivalent? Justify your answer.

What does this tell you about the numbers we can substitute for x?

Let's Practice!

3. Consider the algebraic equation $x + 3 = 9$.

   a. What value can we substitute for x to make it a true number sentence?

   b. How many values could we substitute for x and have a true number sentence?

4. Consider the algebraic equation $x + 6 = x + 9$. What values could we substitute for x to make it a true number sentence?

Try It!

5. Complete the following sentences.

   a. $d^2 = 4$ is true for _____.

   b. $2m = m + m$ is true for _____.

   c. $d + 67 = d + 68$ is true for _____.

BEAT THE TEST!

1. Which of the following equations have the correct solution? Select all that apply.

   - ☐ $2x + 5 = 19;\ x = 7$
   - ☐ $3 + x + 2 - x = 16;\ x = 3$
   - ☐ $\frac{x+2}{5} = 2;\ x = 8$
   - ☐ $6 = 2x - 8;\ x = 7$
   - ☐ $14 = \frac{1}{3}x + 5;\ x = 18$

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Section 2 – Topic 2
Identifying Properties When Solving Equations

The following equations are equivalent. Describe the operation that occurred in the second equation.

$3 + 5 = 8$ and $3 + 5 - 5 = 8 - 5$

$x - 3 = 7$ and $x - 3 + 3 = 7 + 3$

$2(4) = 8$ and $\frac{2(4)}{2} = \frac{8}{2}$

$\frac{x}{2} = 3$ and $2 \cdot \frac{x}{2} = 2 \cdot 3$

This brings us to some more properties that we can use to write equivalent equations.

Properties of Equality

If x is a solution to an equation, then x will also be a solution to the new equation formed when the same number is added to each side of the original equation.

These are the ***addition and subtraction properties of equality***.

- If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- Give examples of this property.

If x is a solution to an equation, x will also be a solution to the new equation formed when each side of the original equation is multiplied by the same number.

These are the ***multiplication and division properties of equality***.

- If $a = b$, then $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$.
- Give examples of this property.

Let's Practice!

1. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $x - 5 = 3x + 7$ and $x - 5 + 5 = 3x + 7 + 5$

   b. $x = 3x + 12$ and $x - 3x = 3x - 3x + 12$

   c. $-2x = 12$ and $\frac{-2x}{-2} = \frac{12}{-2}$

Try It!

2. The following pairs of equations are equivalent. Determine the property that was used to write the second equation.

   a. $2(x+4) = 14 - 6x$ and $2x + 8 = 14 - 6x$

   b. $2x + 8 = 14 - 6x$ and $2x + 8 + 6x = 14 - 6x + 6x$

   c. $2x + 8 + 6x = 14$ and $2x + 6x + 8 = 14$

   d. $8x + 8 = 14$ and $8x + 8 - 8 = 14 - 8$

   e. $8x = 6$ and $\frac{1}{8} \cdot 8x = \frac{1}{8} \cdot 6$

BEAT THE TEST!

1. For each algebraic equation, select the property or properties that could be used to solve it.

Algebraic Equation	Addition or Subtraction Property of Equality	Multiplication or Division Property of Equality	Distributive Property	Commutative Property
$\frac{x}{2} = 5$	☐	☐	☐	☐
$2x + 7 = 13$	☐	☐	☐	☐
$4x = 23$	☐	☐	☐	☐
$x - 3 = -4$	☐	☐	☐	☐
$4(x + 5) = 40$	☐	☐	☐	☐
$10 + x = 79$	☐	☐	☐	☐
$-8 - x = 19$	☐	☐	☐	☐
$2(x - 8) + 7x = 9$	☐	☐	☐	☐

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Section 2 – Topic 3
Solving Equations

Sometimes you will be required to justify the steps to solve an equation. The following equation is solved for x. Use the properties to justify the reason for each step in the chart below.

Statements	Reasons
a. $5(x+3)-2=2-x+9$	**a.** Given
b. $5x+15-2=2-x+9$	**b.**
c. $5x+15-2=2+9-x$	**c.**
d. $5x+13=11-x$	**d.** Equivalent Equation
e. $5x+13-13=11-13-x$	**e.**
f. $5x=-2-x$	**f.** Equivalent Equation
g. $5x+x=-2-x+x$	**g.**
h. $6x=-2$	**h.** Equivalent Equation
i. $\frac{6x}{6}=\frac{-2}{6}$	**i.**
j. $x=-\frac{1}{3}$	**j.** Equivalent Equation

Other times, a word problem or situation may require you to write and solve an equation.

A class is raising funds to go ice skating at the Rink at Campus Martius in Detroit. The class plans to rent one bus. It costs $\$150.00$ to rent a school bus for the day, plus $\$11.00$ per student for admission to the rink, including skates.

What is the variable in this situation?

Write an expression to represent the amount of money the school needs to raise.

The class raised $\$500.00$ for the trip. Write an equation to represent the number of students who can attend the trip.

Solve the equation to determine the number of students who can attend the trip.

Let's Practice!

1. Consider the equation $2x - 3(2x - 1) = 3 - 4x$. Solve the equation for x. For each step, identify the property used to write an equivalent equation.

STUDY EDGE TIP

Some equations, such as $2x = 2x$, have ***all real numbers*** as the solution. No matter what number we substitute for x, the equation will still be true.

Try It!

2. Consider the equation $3(4x + 1) = 3 + 12x - 5$. Solve the equation for x. For each step, identify the property used to convert the equation.

STUDY EDGE TIP

Some equations, such as $2x + 5 = 2x - 1$, have ***no solution***. There is no number that we could substitute for x that will make the equation true.

3. Brooklyn Technical High School surveyed its students about their favorite sports. The 487 students who listed soccer as their favorite sport represented 17 fewer students than three times the number of students who listed basketball as their favorite sport. Write and solve an equation to determine how many students listed basketball as their favorite sport.

BEAT THE TEST!

1. The following equation is solved for x. Use the properties to justify the reason for each step in the chart below.

Statements	Reasons
a. $2(x+5)-3=15$	**a.** Given
b. $2x+10-3=15$	**b.**
c. $2x+7=15$	**c.** Equivalent Equation
d. $2x+7-7=15-7$	**d.**
e. $2x=8$	**e.** Equivalent Equation
f. $\frac{2x}{2}=\frac{8}{2}$	**f.**
g. $x=4$	**g.** Equivalent Equation

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Section 2 – Topic 4
Solving Equations Using the Zero Product Property

If someone told you that the product of two numbers is 10, what could you say about the two numbers?

If someone told you that the product of two numbers is zero, what could you say about the two numbers?

This is the ***zero product property***.

- If $ab=0$, then either $a=0$ or $b=0$.

Describe how to use the zero product property to solve the equation $(x-3)(x+9)=0$. Then, identify the solutions.

Let's Practice!

1. Identify the solution(s) to $2x(x+4)(x+5)=0$.

2. Identify the solution(s) to $(2x-5)(x+11)=0$.

Try It!

3. Michael was given the equation $(x+7)(x-11)=0$ and asked to find the zeros. His solution set was $\{-11,7\}$. Explain whether you agree or disagree with Michael.

4. Identify the solution(s) to $2(y-3)\cdot 6(-y-3)=0$.

BEAT THE TEST!

1. Use the values below to determine the solutions for each equation.

0	2	3	$\frac{4}{5}$
$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-14
6	0	$-\frac{1}{4}$	-2

$(2y+1)(y+14)=0$			
$(7n-2)(5n-4)=0$			
$(4x+3)(x-6)=0$			
$x(x+2)(x-3)=0$			
$t(4t+1)(t-2)=0$			

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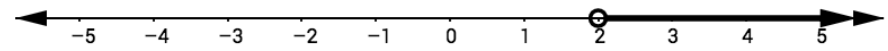
Section 2 – Topic 5
Solving Inequalities – Part 1

Let's start by reviewing how to graph inequalities.



- When the endpoint is a(n) _____ dot or circle, the number represented by the endpoint _____ _____ a part of the solution set.

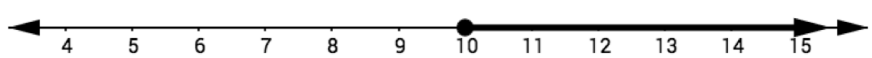
Describe the numbers that are graphed in the example above.

Can you list all the numbers graphed in the example above? Explain your answer.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Consider the following graph.



- When the endpoint is a(n) _____ dot or circle, the number represented by the endpoint _____ a part of the solution set.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Why is "or equal to" included in the solution set?

Just like there are properties of equality, there are also ***properties of inequality***.

If $x > 5$, is $x + 1 > 5 + 1$? Substitute values for x to justify your answer.

Addition and Subtraction Property of Inequality

- If $a > b$, then $a + c > b + c$ and $a - c > b - c$ for any real number c.

Consider $(2x - 1) + 2 > x + 1$. Use the addition or subtraction property of inequality to solve for x.

Let's Practice!

1. Consider the inequality $(4 + x) - 5 \geq 10$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

Try It!

2. Consider the inequality $4x + 8 < 1 + (2x - 5)$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

3. Peter deposited \$27 into his savings account, bringing the total to over \$234. Write and solve an inequality to represent the amount of money in Peter's account before the \$27 deposit.

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Section 2 – Topic 6
Solving Inequalities – Part 2

Consider $x > 5$ and $2 \cdot x > 2 \cdot 5$. Identify a solution to the first inequality. Show that this solution also makes the second inequality true.

Consider $x > 5$ and $-2 \cdot x > -2 \cdot 5$. Identify a solution to the first inequality. Show that this solution makes the second inequality false.

How can we change the second inequality so that the solution makes it true?

Consider $-q > 5$. Use the addition and/or subtraction property of inequality to solve.

Multiplication Property of Inequality

- If $a > b$, then for any positive real number k, ak _____ bk.
- If $a < b$, then for any positive real number k, ak _____ bk.
- If $a > b$, then for any negative real number k, ak _____ bk.
- If $a < b$, then for any negative real number k, ak _____ bk.

The same property is true when dealing with ≤ or ≥.

Let's Practice!

1. Find the solution set of each inequality. Express the solution in set notation and graphically on a number line.

   a. $-9y + 4 < -7y - 2$

   b. $\frac{m}{3} + 8 \leq 9$

2. At 5:00 PM in Atlanta, Georgia, Ethan noticed the temperature outside was 72°F. The temperature decreased at a steady rate of 2°F per hour. At what time was the temperature below 64°F?

Try It!

3. Find the solution set to the inequality. Express the solution in set notation and graphically on a number line.

   a. $-6\,(x-5) > 42$

   b. $4(x+3) \geq 2(2x-2)$

BEAT THE TEST!

1. Ulysses is spending his vacation in South Carolina. He rents a car and is offered two different payment options. He can either pay $\$25.00$ each day plus $\$0.15$ per mile (option A) or pay $\$10.00$ each day plus $\$0.40$ per mile (option B). Ulysses rents the car for one day.

   Part A: Write an inequality representing the number of miles where option A will be the cheaper plan.

   Part B: How many miles will Ulysses have to drive for option A to be the cheaper option?

2. Stephanie has just been given a new job in the sales department of Frontier Electric Authority. She has two salary options. She can either receive a fixed salary of $\$500.00$ per week or a salary of $\$200.00$ per week plus a 5% commission on her weekly sales. The variable s represents Stephanie's weekly sales. Which solution set represents the dollar amount of sales that she must generate in a week in order for the option with commission to be the better choice?

   Ⓐ $\{s|s > \$300.00\}$
   Ⓑ $\{s|s > \$700.00\}$
   Ⓒ $\{s|s > \$3{,}000.00\}$
   Ⓓ $\{s|s > \$6{,}000.00\}$

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Section 2 – Topic 7
Solving Compound Inequalities

Consider the following options.

Option A: You get to play *NBA 2K* after you clean your room and do the dishes.

Option B: You get to play *NBA 2K* after you clean your room or do the dishes.

What is the difference between Option A and B?

Circle the statements that are true.

$2 + 9 = 11$ and $10 < 5 + 6$

$4 + 5 \neq 9$ and $2 + 3 > 0$

$0 > 4 - 6$ or $3 + 2 = 6$

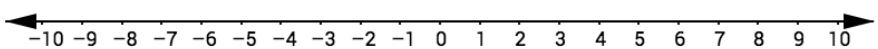
$15 - 20 > 0$ or $2.5 + 3.5 = 7$

These are called ***compound equations*** or ***inequalities***.

- When the two statements in the previous sentences were joined by the word *AND*, the compound equation or inequality is true only if _____ statements are true.

- When the two statements in the previous sentences were joined by the word *OR*, the compound equation or inequality is true if at least _____ of the statements is true. Therefore, it is also considered true if _____ statements are true.

Let's graph $x < 6$ and $x > 1$.



This is the _____ _____ to the compound inequality.

How many solutions does this inequality have?

Many times this is written as $1 < x < 6$. This notation denotes the conjunction "and."

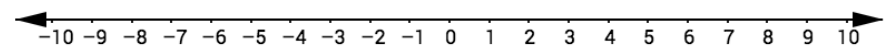
We read this as "x is greater than one _____ less than six."

Let's Practice!

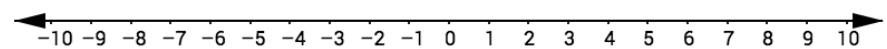
1. Consider $x < 1$ or $x > 6$. Could we write the inequalities above as $1 > x > 6$? Explain your answer.

2. Graph the solution set to each compound inequality on a number line.

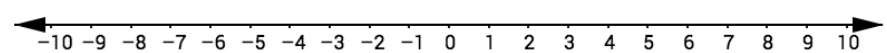
   a. $x = 2$ or $x > 5$



   b. $x > 6$ or $x < 6$



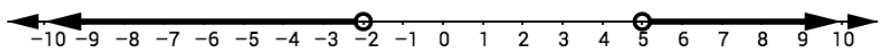
   c. $1 \leq -x \leq 7$



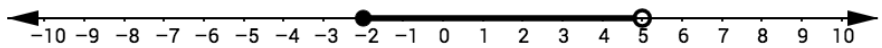
STUDY EDGE TIP

Be on the lookout for negative coefficients. When solving inequalities, you will need to reverse the inequality symbol when you multiply or divide by a negative value.

3. Write a compound inequality for the following graphs.



a. Compound inequality: _____

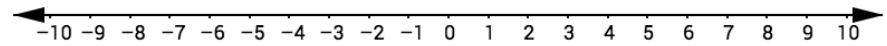


b. Compound inequality: _____

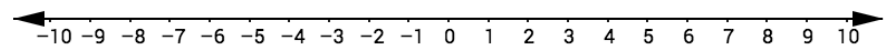
Try It!

4. Graph the solution set to each compound inequality on a number line.

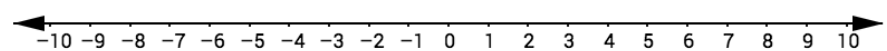
a. $x < 1$ or $x > 8$



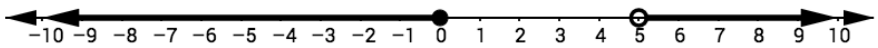
b. $x \geq 6$ or $x < 4$



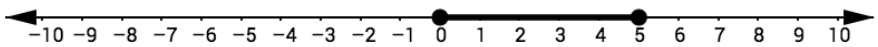
c. $-6 \leq x \leq 4$



5. Write a compound inequality for the following graphs.



a. Compound inequality: _____

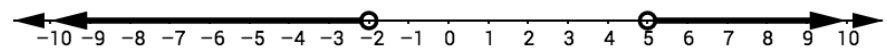


b. Compound inequality: _____

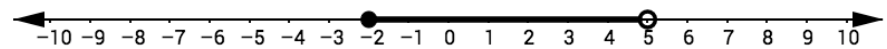
BEAT THE TEST!

1. Use the terms and symbols in the table to write a compound inequality for each of the following graphs. You may only use each term once, but you do not have to use all of them.

$3x$	-14	-6	$\geq$	$-$	17	15	$<$
$7x$	$<$	2	or	$\leq$	$3x$	$+$	$>$



Compound Inequality: _____



Compound Inequality: _____

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Section 2 – Topic 8
Rearranging Formulas

Solve each equation for x.

$2x + 4 = 12$ $2x + y = z$

Did we use different properties when we solved the two equations?

Consider the formula for the perimeter of a rectangle: $P = 2l + 2w$.

Sometimes, we might need the formula solved for length.

STUDY EDGE TIP

When solving for a variable, it's helpful to circle that variable.

Let's Practice!

1. Consider the equation $rx - sx + y = z$; solve for x.

Try It!

2. Consider the equation $8c + 6j = 5p$; solve for c.

3. Consider the equation $\frac{x-c}{2} = d$; solve for c.

BEAT THE TEST!

1. Isaiah planted a seedling in his garden and recorded its height every week. The equation shown can be used to estimate the height, h, of the seedling after w weeks since he planted the seedling.

$$h = \frac{3}{4}w + \frac{9}{4}$$

Solve the formula for w, the number of weeks since he planted the seedling.

2. Under the Brannock device method, shoe size and foot length for women are related by the formula $S = 3F - 21$, where S represents the shoe size and F represents the length of the foot in inches. Solve the formula for F.

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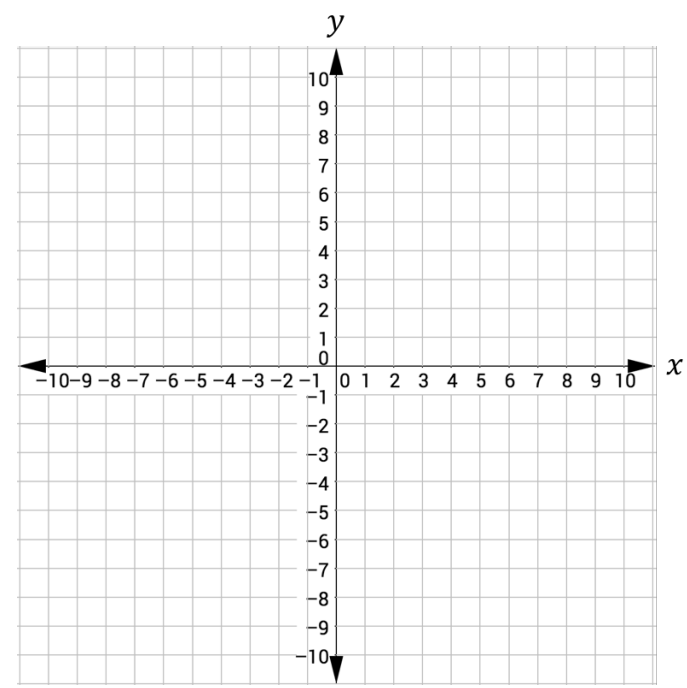
Section 2 – Topic 9
Solution Sets to Equations with Two Variables

Consider $x + 2 = 5$. What is the only possible value of x that makes the equation a true statement?

Now consider $x + y = 5$. What are some solutions for x and y that would make the equation true?

Possible solutions can be listed as ***ordered pairs***.

Graph each of the ordered pairs from the previous problem on the graph below.



What do you notice about the points you graphed?

How many solutions are there to the equation $x + y = 5$?

Let's Practice!

1. Taylor has 10 songs on her phone's playlist. The playlist features songs from her two favorite artists, Beyoncé and Pharrell.

   a. Create an equation using two variables to represent this situation.

   b. List at least three solutions to the equation that you created.

   c. Does this equation have infinitely many solutions? Why or why not?

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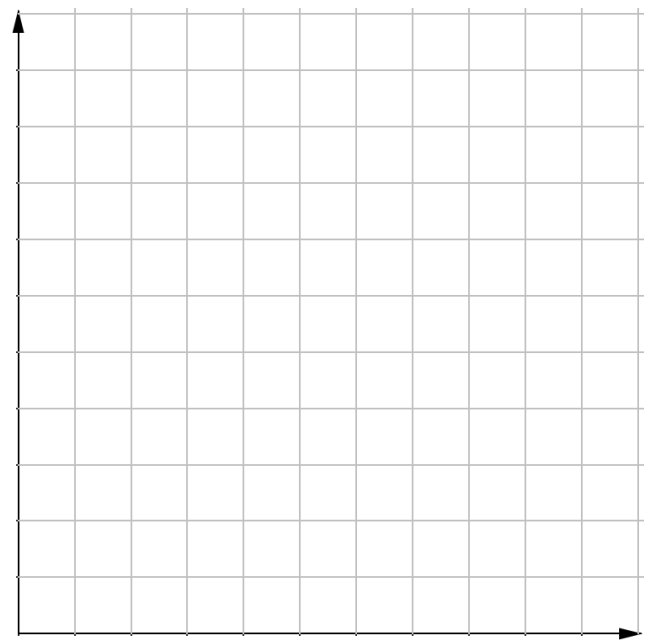
In this case, our solutions must be natural numbers. This is called a ***discrete function***. Notice that the solutions follow a linear pattern. However, they do not form a line.

d. Create a graph that represents the solution set to your equation.



e. Why are there only positive values on this graph?

Try It!

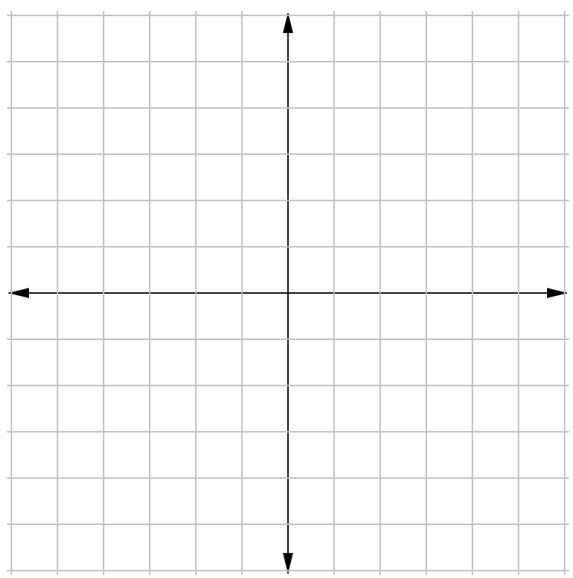
2. The sum of two numbers is 15.

   a. Create an equation using two variables to represent this situation.

   b. List at least three possible solutions.

   c. How many solutions are there to this equation?

   d. Create a visual representation of all the possible solutions on the graph.



STUDY EDGE TIP

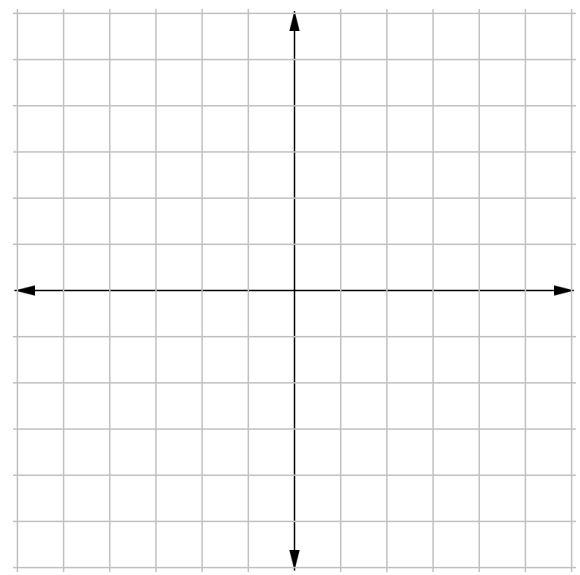
In this case, we have a ***continuous function***. Notice the solutions are rational numbers and they form a line.

3. What if we changed the problem to say the sum of two integers is 15?

   a. Create an equation using two variables to represent this situation.

   b. Is this function discrete or continuous? Explain your answer.

   c. Represent the solution on the graph below.



BEAT THE TEST!

1. Elizabeth's tablet has a combined total of 20 apps and movies. Let x represent the number of apps and y represent the number of movies. Which of the following could represent the number of apps and movies on Elizabeth's tablet? Select all that apply.

   - ☐ $x + y = 20$
   - ☐ 7 apps and 14 movies
   - ☐ $x - y = 20$
   - ☐ $y = -x + 20$
   - ☐ 8 apps and 12 movies
   - ☐ $xy = 20$

Test Yourself! Practice Tool

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!

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